The probability that the client can connect after $k$ tries is:

$$P(\text{connect after } k \text{ tries}) = 1 - (1 - P(\text{connect after } 1 \text{ try}))^k = 1 - (1 - (1 - \epsilon_i)^i)^k$$

the required number of connection attempts is:

$$k = \frac{\log(1 - P(\text{connect}))}{\log(1 - (1 - \epsilon_i)^i)}$$

A nice feature of this formula is that the expected number of connection attempts depends logarithmically on the connection probability, which indicates that even for large $\epsilon_i$, a determined client can get a connection after a moderate waiting time.